

## 石家庄铁道大学应用数学和计算数学研讨会日程

2019年4月20日，星期六，地点：数理系会议室

主持人：李鹏	8:30-8:50	开幕式，李向红副主任致欢迎词，集体合影
主持人：高振	8:50-9:30	Wai-Sun Don（中国海洋大学） An alternative characteristic-wise WENO-Z finite difference scheme for solving the compressible multi-component non-reactive flows in the overestimated quasi-conservative form
	9:30-10:10	李宏伟（山东师范大学） Unconditionally energy stable linear schemes for the diffuse interface model with Peng-Robinson equation of state
	10:10-10:30	茶歇
主持人：李宏伟	10:30-11:10	高振（中国海洋大学） A Non-intrusive Reduced Basis Method for Rayleigh-Taylor instability problem
	11:10-11:50	左大伟（石家庄铁道大学） 水波中的孤子
	11:50-14:00	午饭及午休时间
主持人：左大伟	14:00-14:30	赵永耀（晋中学院） 可燃气体爆燃转爆轰的大涡模拟研究
	14:30-15:00	温晓（中国海洋大学） Entropy Stable and Well-Balanced Discontinuous Galerkin Methods for the Nonlinear Shallow Water Equations
	15:00-15:30	李鹏（石家庄铁道大学） High Order Well-Balanced WENO Interpolation-Based Finite Difference Schemes for Shallow Water Equations
	15:30-15:50	茶歇
主持人：赵永耀	15:50-16:20	王保山（中国海洋大学） Hybrid compact-WENO finite difference scheme with radial basis function based shock detection method for hyperbolic conservation laws
	16:20-16:50	王英华（中国海洋大学） Generalized Sensitivity Parameter Free Fifth Order WENO Finite Difference Scheme with Z-Type Weights

2019年4月21日，星期日，地点：数理系会议室

主持人： Wai-Sun Don	8:30-10:30	学术讨论
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**An alternative characteristic-wise WENO-Z finite difference scheme for solving the compressible multicomponent non-reactive flows in the overestimated quasi-conservative form**

Wai Sun Don

School of Mathematical Sciences, Ocean University of China, Qingdao, China.

Abstract: The fifth, seventh and ninth order characteristic-wise alternative weighted essentially non-oscillatory (AWENO) finite difference schemes are applied to the fully conservative (FC) form and overestimated quasi-conservative (OQC) form of the compressible multicomponent flows. Several linear and nonlinear numerical operators such as the linear Lax-Friedrichs operator and linearized nonlinear WENO operator and their mathematical properties are defined to build a mathematical framework for identifying conditions required in maintaining the equilibriums. In the case of OQC form, the AWENO scheme with the modified flux can be rigorously proved to maintain the equilibriums of velocity, pressure and temperature. Furthermore, we also show that the FC form cannot maintain the equilibriums without an additional advection equation of auxiliary variable involving the specific heat ratio. Extensive one- and two-dimensional classical problems such as the moving material interface problem, multifluid shock-density interaction problem and shock-R22-bubble interaction problem verify the theoretical results and also show that the AWENO scheme demonstrates less dissipation errors and higher resolution than the classical WENO-Z scheme in the splitting form [T. Nonomura et al. J. Comput. Phys. 340 (2017)].

This research is a joint work with Dong-Mei Li, Zhen Gao, Bao-Shan Wang at the School of Mathematical Sciences of Ocean University of China

**Unconditionally energy stable linear schemes for the diffuse interface model with Peng-Robinson equation of state**

Hongwei Li

School of Mathematics and Statistics, Shandong Normal University, Jinan, China

Abstract: In this paper, we investigate numerical solution of the diffuse interface model with Peng-Robinson equation of state, that has been widely used to describe real states of hydrocarbon fluids in the petroleum industry. Due to the strong nonlinearity of the source terms in this model, how to design appropriate time discretizations to preserve the energy dissipation law of the system at the discrete level is a major challenge. Based on the “Invariant Energy Quadraticization” approach, we develop a first and a second order time stepping schemes for solving the single-component two-phase fluid problem. In both schemes the resulted temporal semi-discretizations lead to linear systems with symmetric positive definite spatial operators at each time step, and thus can be efficiently solved. We also rigorously prove the unconditional energy stability of both schemes. Various numerical simulations in two and three dimensional spaces are presented to demonstrate accuracy and stability of the proposed linear schemes and to investigate physical reliability of the target model by comparisons with laboratory data.

## A Non-intrusive Reduced Basis Method for Rayleigh-Taylor instability problem

Zhen Gao

School of Mathematical Sciences, Ocean University of China, Qingdao, China.

**Abstract:** Rayleigh-Taylor instability (RTI) phenomenon plays an important role in the confined fusion implosion etc. However, its simulations heavily depend on the initial perturbation waves with different amplitudes which result in a parametrized PDEs. In this talk, a non-intrusive reduced basis method is introduced to solve the parametrized PDEs efficiently and simulate RTI with initial arbitrary amplitudes at any time. The detailed algorithms of finding the reduced basis, corresponding coefficients and sampling methods are also reviewed. The simulated results are comparable to those computed by the PDE solver.

## 可燃气体爆燃转爆轰的大涡模拟研究

赵永耀

晋中学院，数理系，山西. 太原

**摘要:** 可燃气体爆燃转爆轰机理是工业爆炸灾害的防治和超高声速推进领域急需解决的关键科学问题。气体爆炸多发生在大尺度复杂空间内，这为数值模拟研究提出了严峻的挑战。大涡模拟 (LES) 计算量比 DNS 小，同时能比 RANS 获得更多的流场信息，因此成为最有发展前景的数值模拟方法。本文主要研究了能够模拟气体爆燃转爆轰整个过程的大涡模拟亚网格模型，包括湍流模型和燃烧模型；同时研究了壁面热传导以及障碍物对火焰加速及爆燃转爆轰的作用。

## Entropy Stable and Well-Balanced Discontinuous Galerkin Methods for the Nonlinear Shallow Water Equations

Xiao Wen

School of Mathematical Sciences, Ocean University of China, Qingdao, China.

**Abstract:** The nonlinear shallow water equations (SWEs) are widely used to model the unsteady water flows in rivers and coastal areas, with extensive applications in ocean and hydraulic engineering. In this work, we propose entropy stable, well-balanced and positivity-preserving discontinuous Galerkin (DG) methods, under more arbitrary choices of quadrature rules, for the SWEs with non-at bottom topography. In “Chan. J Comput Phys. 362:346-374, 2018”, a SBP-like differentiation operator is introduced to construct the discretely entropy conservative DG methods. We extend this idea to the SWEs and establish an entropy stable scheme by adding additional dissipative terms. Careful approximation of the source term is included to ensure the well-balanced property of the resulting method. A simple positivity-preserving limiter, compatible with the entropy stable property, is added to guarantee the non-negative water heights during the computation. One- and two-dimensional numerical experiments are presented to demonstrate the performance of the proposed methods.

## **High Order Well-Balanced WENO Interpolation-Based Finite Difference Schemes for Shallow Water Equations**

Peng Li

Department of Mathematics and Physics, Shijiazhuang Tiedao University, Shijiazhuang, China.

**Abstract:** In this paper, a numerical framework of the general form of high order well-balanced WENO interpolation-based finite difference schemes based on the high order WENO interpolation and arbitrary numerical monotone flux are proposed for the shallow water equations. We employ a special splitting technique for the source term proposed in [Y. Xing, C.-W Shu, J. Comput. Phys. 208 (2005) 206-227] and the reconstruct techniques for the conservative variables in [Y. Xing and C.-W Shu, J. Comput. Phys. 214 (2006) 567-598] to maintain the exact C-property, which can be proved theoretically. In the meantime, the genuine high order accuracy of the numerical scheme can be observed successfully, and small perturbation of the stationary state can be resolved and evolved well. In order to capture the strong discontinuities and large gradients, the 5th-order upwind weighted nonlinear interpolations for the conservative variables combining the high order compact schemes and high order correction terms are employed. In addition, the local characteristic projections are considered to further restrain the potential numerical oscillations. A variety of representative one- and two-dimensional examples are tested to demonstrate the good performance of the proposed schemes.

## **Hybrid compact-WENO finite difference scheme with radial basis function based shock detection method for hyperbolic conservation laws**

Bao-Shan Wang

School of Mathematical Sciences, Ocean University of China, Qingdao.

**Abstract:** Hybrid scheme, based on the high order nonlinear characteristic-wise weighted essentially non-oscillatory (WENO) conservative finite difference scheme and the spectral-like linear compact finite difference scheme, has been developed for capturing shocks and strong gradients accurately and resolving fine scale structures efficiently for hyperbolic conservation laws. The key issue in any hybrid scheme is the design of an accurate, robust, and efficient high order shock detection algorithm which is capable of determining the smoothness of the solution at any given grid point and time. An improved iterative adaptive multi-quadric radial basis function (IAMQ-RBF-Fast) method [Don et al. J. Sci. Comput. 75(2) (2018) 1016-1039], which employed the  $O(N^2)$  recursive Levinson-Durbin method and the Sherman-Morrison-Woodbury method for solving the perturbed Toeplitz matrix system, has been successfully developed as an efficient and accurate edge detector of the piecewise smooth functions. In this study, the method, together with the Tukey's boxplot method and the domain segmentation technique, is extended to serve as a novel shock detection algorithm for solving the Euler equations. The applicability and performance of the RBF edge detection method as the shock detector in the hybrid scheme in terms of accuracy, robustness, efficiency, resolution and other implementation issues are given. Several one- and two-dimensional benchmark problems in shocked flow demonstrate that the proposed hybrid scheme can reach a speedup of the CPU times by a factor up to 2-3 compared with the pure fifth order WENO-Z scheme.

# Generalized Sensitivity Parameter Free Fifth Order WENO Finite Difference Scheme with Z-Type Weights

Yinghua Wang

School of Mathematical Sciences, Ocean University of China, Qingdao.

**Abstract:** A modified fifth order Z-type (nonlinear) weights, which consist of a linear term and a nonlinear term, in the weighted essentially non-oscillatory (WENO) polynomial reconstruction procedure for the WENO-Z finite difference scheme is proposed. The nonlinear term is modified by a modifier function that is based on the linear combination of the local lower order smoothness indicators. The new WENO weights with the modified Z-type weights (WENO-D) scheme and its improved version (WENO-A) scheme are proposed. This enhancement unifies many existing Z-type weights based WENO schemes. This class of WENO schemes with Z-type weights are analyzed for the error and order of accuracy for approximating the derivative of a smooth function with high order critical points. The analysis and numerical experiments show that, the proposed WENO schemes achieve the optimal (fifth) order of accuracy regardless of the order of critical point with any sensitivity parameter. With a properly chosen variable sensitivity parameter, the proposed WENO schemes also allow a quicker convergence and a significant error reduction over the WENO-Z scheme at a coarse mesh resolution (under-resolved). The proposed WENO schemes achieve an improved balance between the linear term and the modified nonlinear term. The performance of the WENO schemes are compared by solving several one- and two-dimensional benchmark shocked flows. The results show that the proposed WENO schemes perform overall as well as, if not slightly better than, the WENO-Z scheme.

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