AWENO-Z scheme for compressible multicomponent flows

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Introduction

The AWENO-Z finite difference scheme is employed to solve the multicomponent Euler equations with the overestimated quasi-conservative (OQC) form that satisfies the equilibriums. From the results of the moving material interface problem, we can see that all the OQC implementations can maintain such equilibriums while the FC implementation fails to do so. Furthermore, for example in the multifluid shock-density interaction problem, the fifth order alternative WENO (OQC-AWENO5) demonstrates a higher resolution and less dissipative solution than the fifth order classical WENO (OQC-WENO5). The seventh and ninth order AWENO schemes with the OQC form of the governing equations in simulating multicomponent flows have a greatly improved resolution and less dissipation than the fifth order one with a given mesh resolution.

AWENO scheme in the OQC form

The overestimated quasi-conservative form of the governing equation is presented as follows:

$$\mathbf{Q}=\begin{bmatrix}\rho\\ \rho u\\ \rho e\\ \rho Y_1\\ \frac{1}{\gamma_p-1}\end{bmatrix},\quad \mathbf{M}=\begin{bmatrix}1&&&&\\&1&&&\\&&1&&\\&&&1&\\&&&&u\end{bmatrix},\quad \mathbf{F}=\begin{bmatrix}\rho u\\ \rho u^2+p\\ u(\rho e+p)\\ \rho u Y_1\\ \frac{1}{\gamma_p-1}\end{bmatrix},$$

$$\frac{\partial \mathbf{Q}}{\partial t}+\mathbf{M}\frac{\partial \mathbf{F}}{\partial x}=0.$$

Here, $\frac{1}{\gamma_p-1}$ is denoted as Γ.

Because we cannot define the flux Jacobian due to its quasi-conservative forms, the eigensystem of the following equation is considered instead:

$$\frac{\partial \mathbf{Q}}{\partial t}+\mathbf{B}\frac{\partial \mathbf{Q}}{\partial x}=0,\ \mathbf{B}=\mathbf{M}\frac{\partial \mathbf{F}}{\partial \mathbf{Q}},\mathbf{B}=\mathbf{R}\Lambda\mathbf{L},$$

where $\mathbf{L}$ and $\mathbf{R}$ are the left and right eigenvectors of $\mathbf{B}$.

The derivative of the fluxes after multiplying by the $\mathbf{M}_j$ is approximated in the conservative manner as

$$\mathbf{M}_j\left(\frac{\partial \mathbf{F}}{\partial x}\right)_j\approx\frac{\hat{\mathbf{F}}^M_{j+\frac12}-\hat{\mathbf{F}}^M_{j-\frac12}}{\Delta x}.$$

The modified numerical flux $\hat{\mathbf{F}}^M_{j\pm\frac12}$ is

$$\hat{\mathbf{F}}^M_{j\pm\frac12}=\mathbf{F}^M_{j\pm\frac12}+\mathbf{M}_j\mathcal{D}_{j\pm\frac12}\mathbf{F},$$

where

$$\mathbf{F}^M_{j\pm\frac12}=\frac12[\mathbf{M}_j\left(\mathbf{F}\left(\mathbf{Q}^+_{j\pm\frac12}\right)+\mathbf{F}\left(\mathbf{Q}^-_{j\pm\frac12}\right)\right)-\alpha\left(\mathbf{Q}^+_{j\pm\frac12}-\mathbf{Q}^-_{j\pm\frac12}\right)].$$

AWENO scheme in the OQC form

1.Interpolation of the conservative variable

$$\mathbf{Q}_{j+\frac12}=\tilde{\mathbf{R}}W_{j+\frac12}[\tilde{\mathbf{L}}\mathbf{Q}_{j+l},\ \ l=-(r-1),...,r].$$

- Projection the conservative variables into the characteristic space

$$\tilde{\mathbf{Q}}_{j+l}=\tilde{\mathbf{L}}\mathbf{Q}_{j+l}=\left[\ 0,\ \ \rho,\ \ 0,\ \ \rho(Y_1-\tilde{Y}_1),\ \ \Gamma\ \right]^T_{j+l}.$$

- Using the WENO interpolation procedure, the reconstructed characteristic variables

$$\tilde{\mathbf{Q}}^{\pm}_{j+\frac12}=W^{\pm}_{j+\frac12}\left[\tilde{\mathbf{Q}}\right]=W^{\pm}_{j+\frac12}\left[\tilde{\mathbf{L}}\mathbf{Q}\right]\triangleq\left[q_1^{\pm},q_2^{\pm},q_3^{\pm},q_4^{\pm},q_5^{\pm}\right]^T_{j+\frac12}.$$

- $\tilde{\mathbf{Q}}^{\pm}_{j+\frac12}$ are projected back into the physical space to form the WENO reconstructed conservative variables

$$\mathbf{Q}^{\pm}_{j+\frac12}=\tilde{\mathbf{R}}\tilde{\mathbf{Q}}^{\pm}_{j+\frac12}=\left(\left[q_2,uq_2,\frac12u^2q_2+Pq_5,\tilde{Y}_1q_2+q_4,q_5\right]^{\pm}_{j+\frac12}\right)^T.$$

- The flux based on the WENO reconstructed conservative variable

$$\mathbf{F}\left(\mathbf{Q}^{\pm}_{j+\frac12}\right)=\begin{bmatrix}uq_2\\ P+u^2q_2\\ Pu+u\left(\frac12u^2q_2+Pq_5\right)\\ u\left(\tilde{Y}_1q_2+q_4\right)\\ q_5\end{bmatrix}^{\pm}_{j+\frac12},$$

Here, $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{R}}$ are the simple-averaged left and right eigenvectors, respectively.

2.High order derivative terms of the AWENO scheme

For the linear approximation of the higher order expansion of the flux $\mathbf{F}$ involving higher order derivatives of $\mathbf{F}$, at the cell boundaries, $\mathcal{D}_{j+\frac12}\mathbf{F}$, up to the ninth order has the form:

$$\mathcal{D}_{j+\frac12}\mathbf{F}=\sum_{k=1}^{r-1}a_{2k}\Delta x^{2k}\mathbf{F}^{(2k)}_{j+\frac12},$$

with the coefficients

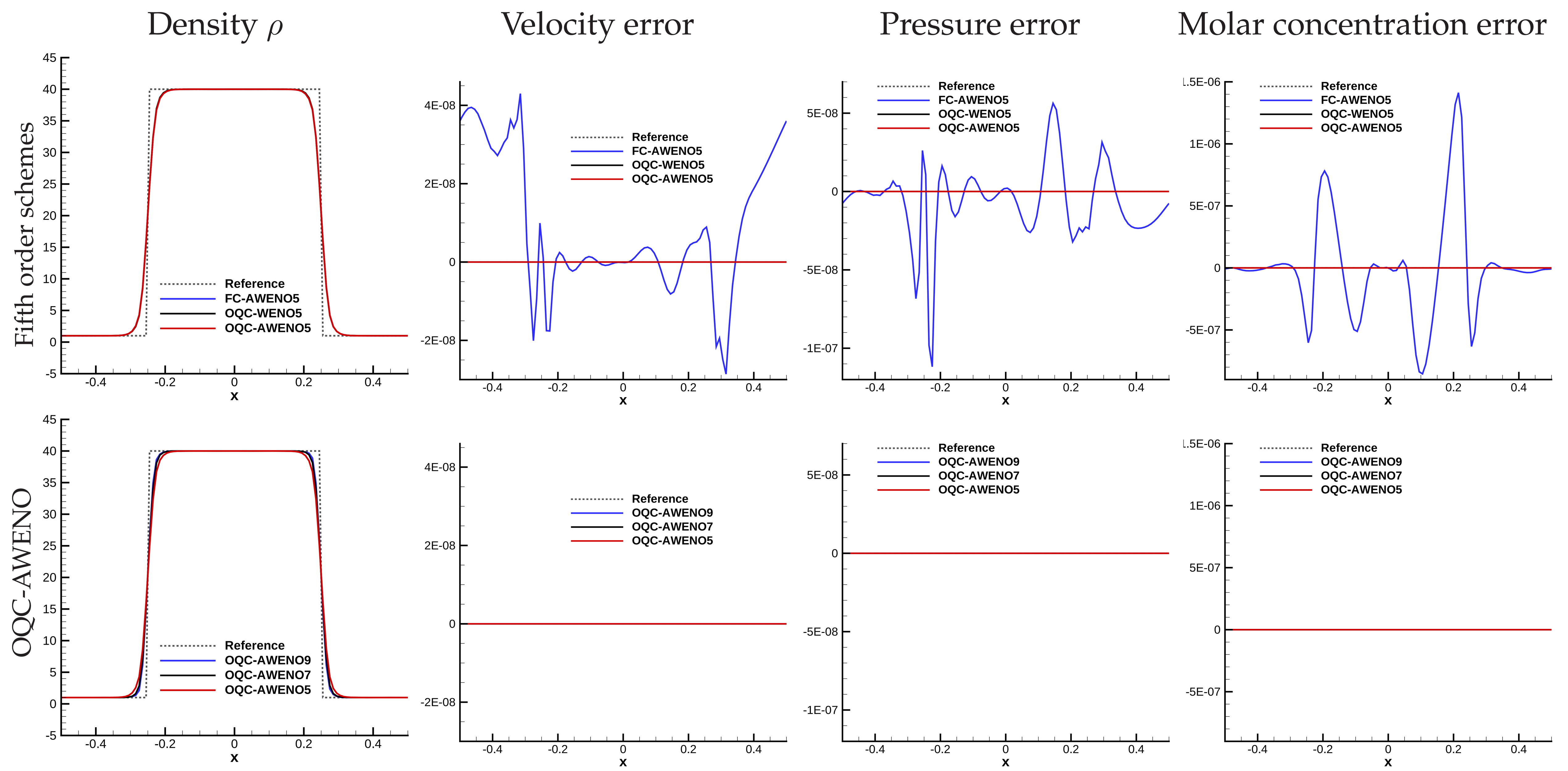
$$a_2=-\frac{1}{24},\ a_4=\frac{7}{5760},\ a_6=-\frac{31}{967680},\ a_8=\frac{127}{154828800}.$$

For a sufficiently smooth function $\mathbf{F}$, the higher order derivative terms $\Delta x^{2k}\mathbf{F}^{(2k)}_{j+\frac12}$ are approximated by the central finite difference scheme with an overall global accuracy of order $O(\Delta x^{2r})$,

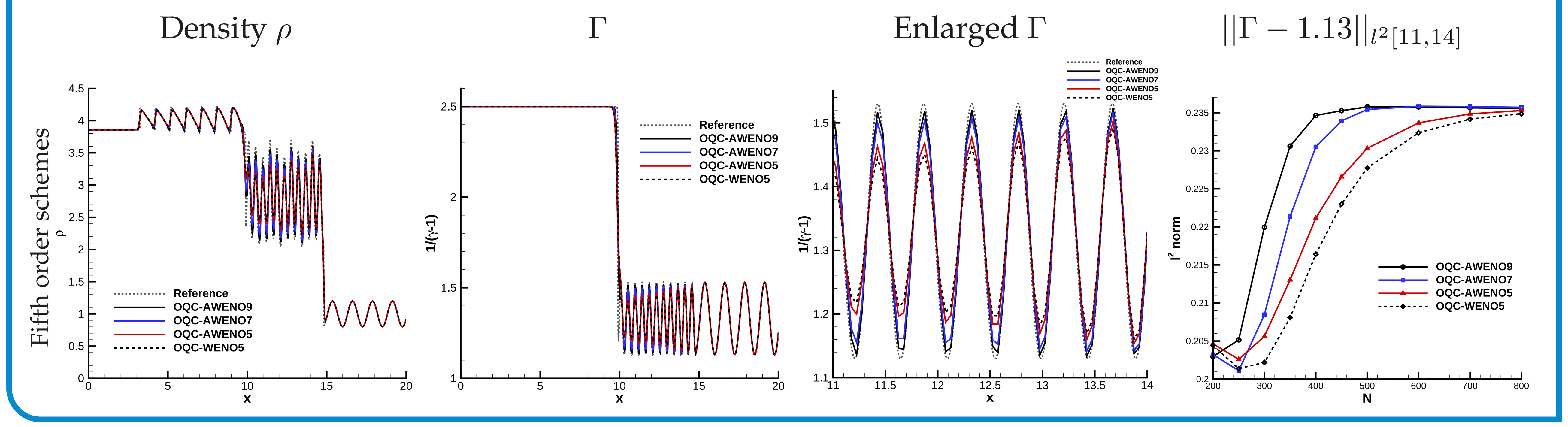
$$\Delta x^{2k}\mathbf{F}^{(2k)}_{j+\frac12}=\sum_{n=1}^{r}d_n^{2k}(\mathbf{F}_{j-n+1}+\mathbf{F}_{j+n}),$$

1D problem

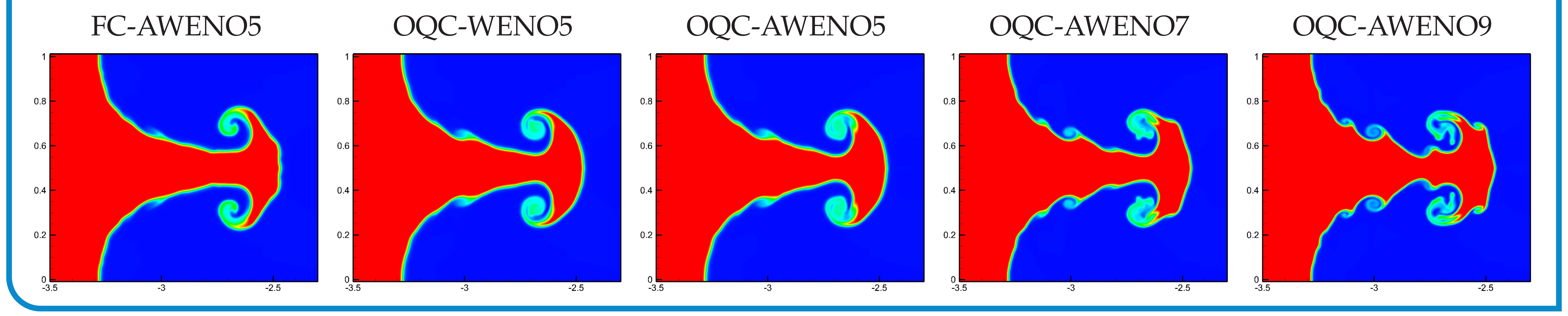
Moving material interface problem ($t=1, N=100$):



Extended multifluid shock-density interaction problem ($t=3.5, N=400$):



2D Richtmyer-Meshkov instability problem($t=8.25$, 2048×256)



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