IAMQ-RBF Method for Shock Detection

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Introduction

Radial basis function (RBF) have gained popularity for many applications including numerical solution of partial differential equations (PDEs), computer graphics (CG) and artificial neural networks (ANNs).

The main advantage of RBF method is that they do not necessarily require any particular grid system consequently yielding more flexibility when approximating the function in an irregular domain. The adaptive RBF method considerably reduces the Gibbs oscillations and yields a fast convergence if the function considered is smooth enough.

In this research, we mainly apply the iterative adaptive multi-quadric RBF (IAMQ-RBF) method to detect shocks in solving Euler equation with Hybrid Compact-WENO scheme.

MQ-RBF Definition

Consider a set X composed of the points x_i, $X=\{x_i|x_i\in\Omega, i=1,2,...,N\}$, and let f denote the set of function values, $f=\{f_i|f_i=f(x_i)\in\mathbb{R}, x_i\in X\}$ for the real-valued function $f(x)$.

In this work, the MQ-RBF $\Psi(x)$ is defined by

$$\Psi_i(x)=\sqrt{(x-x_i)^2+\epsilon_i^2}\quad x_i\in X,\ x\in\Omega \tag{1}$$

where ϵ_i are the shape parameters.

Define interpolation matrix M, derivative operator D, expansion coefficient vector λ and concentration set C as follows:

$$M_{ij}=\sqrt{(x_i-x_j)^2+\epsilon_j^2} \tag{2}$$

$$D_{ij}=(x_i-x_j)/M_{ij},\quad \text{if } \epsilon_j\neq 0 \tag{3}$$

$$D_{jj}=0,\quad \text{if } \epsilon_j=0 \tag{4}$$

$$\lambda=M^{-1}f \tag{5}$$

$$C_i=|\lambda_i(D\lambda)_i| \tag{6}$$

where $i,j=1,2,...,N$.

IAMQ-RBF Algorithm

Given: $\{\epsilon_i\}$, $\eta>0$, $\delta>0$, $\text{flag}_i=0$

Step 1: Compute and normalize C, using Eq.(6) with $\{\epsilon_i\}$

Step 2: Find $\mathbf{S}$, $\mathbf{S}=\{x_i|\ x_i\in X,\ C_i\geq\eta>0\}$

Step 3: Update ϵ_i, let $\epsilon_i=0$ at $x_i\in\mathbf{S}$

Step 4: Repeat Step 1 through Step 3 if $\|\lambda^{\text{new}}-\lambda^{\text{old}}\|>\delta$

Step 5: $\text{flag}_i=1$ if $\epsilon_i=0$

Remark: The algorithm bases on ϵ_i, adaptation criteria η and iteration tolerance level δ.

For the stability, X should be mapped to $[-x,x]$ before iteration if $Range(X)<2x_c$, where $x\geq x_c=0.006N-0.2$. In this work, we choose $x=0.006N$.

Hybrid Scheme

By performing IAMQ-RBF algorithm of a given variable (usally density ρ), a falg is specified by

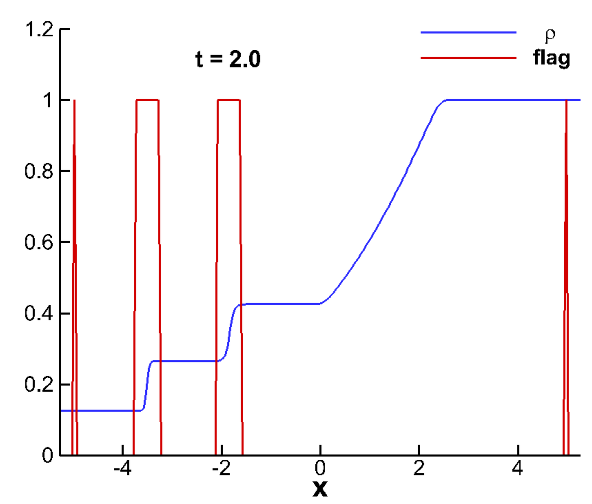
$$\text{flag}_i=\begin{cases}1, & \epsilon_i=0,\quad \text{Non-Smooth,}\\ 0, & \epsilon_i\neq 0,\quad \text{Smooth.}\end{cases} \tag{7}$$

A buffer zone is created around each grid point that is flagged as non-smooth with $\text{flag}_i=1$. The Euler equation

$$\mathbf{Q}_t+\nabla\cdot\mathbf{F}=\mathbf{0} \tag{8}$$

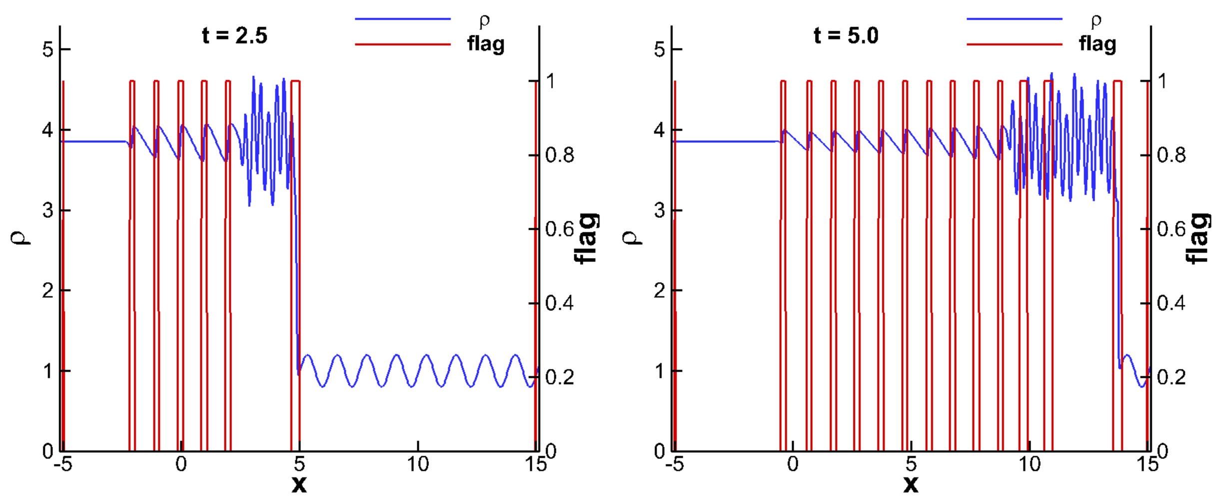
is solved by the WENO-Z and compact schemes at non-smooth and smooth stencils respectively. In smooth stencils, the fluxes on the non-smooth zones obtained by the fifth-order WENO-Z scheme are automatically used as the internal boundary fluxes for the sixth-order compact scheme. And a sixth-order filtering is applied in the smooth stencils to stabilize the Hybrid scheme. In time, we use third-order TVD Runge-Kutta scheme with CFL $=0.45$.

Sod Problem Analysis



As shown in this figure, the two stronger shocks at time $t=2.0$, can be identified by setting $\epsilon_i=0.07, \eta=0.5, \delta=10^{-10}$. However, the two weaker shocks are missed.

Shu-Osher Problem Analysis



In this problem ($N=800$), the method detects the strongest shock and other shocks successfully.

The efficiency of IAMQ-RBF alogrithm will be reduced with N becoming larger because it will take more time to solve the system of linear equations $M\lambda=f$. Here, we shall use the domain decomposition method (see below) for large N.

Improved Algorithm and Shu-Osher Problem Analysis

We decompose the domain into smaller sub-domains and then use IAMQ-RBF to detect shocks in each sub-domain. The final flag is the union of the flag from each sub-domain.

There are 4 overlapping sub-domains decomposed from X in Fig.1.

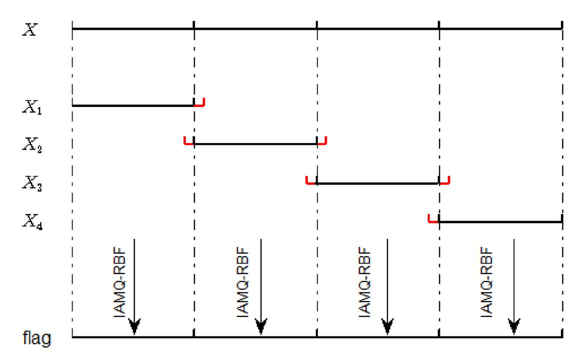


Figure 1: Domain decomposition method

Remark: Some points near boundary will be detected by this method if the function is smooth in some sub-domain. For reducing this phenomenon, the IAMQ-RBF algorithm should be changed at **Step 5** as follows:

$$\text{flag}_i=1,\ \text{if } \epsilon_i=0\ \&\ |\lambda_i|>\lambda_0$$

where $\lambda_0=0.5$.

Consider Shu-Osher problem ($N=800$).

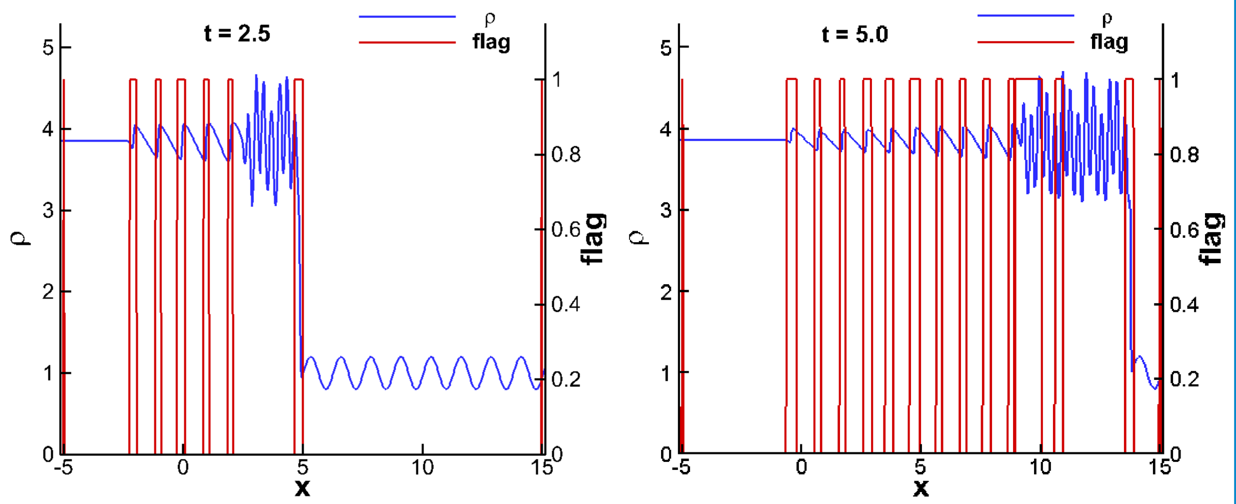


Figure 2: Using 4 sub-domains and 20 overlapping points.

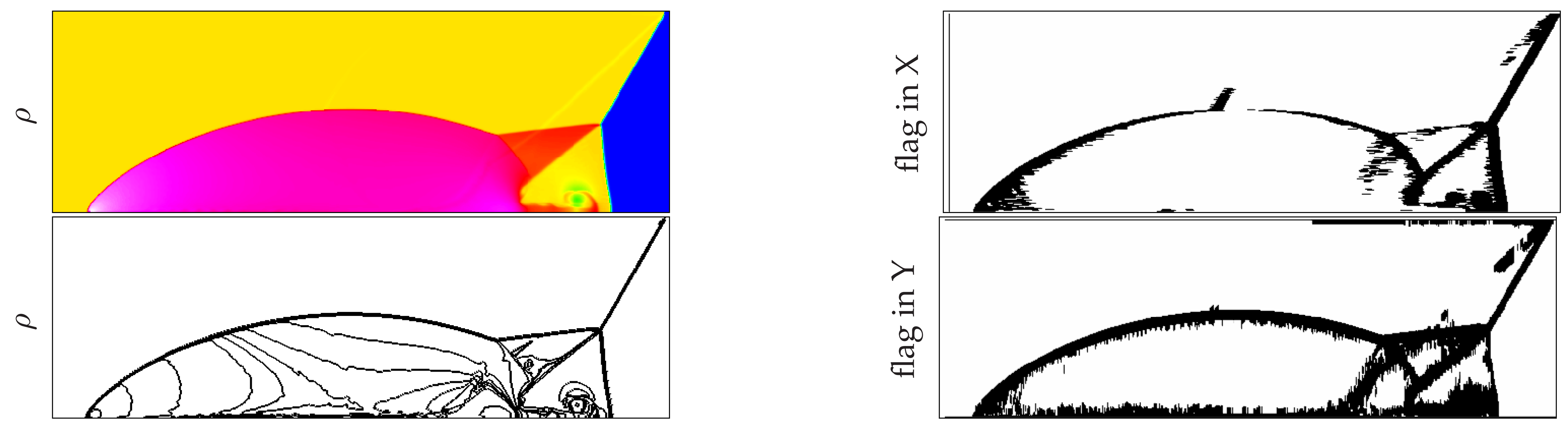
Table 1: CPU Time

sub-domains	CPU time(s)
1	362.8
2	187.4
4	82.56
8	73.77

As shown in table 1 and Fig.2, the improved method is faster than before. However, it finds out more false points in where ρ is oscillatory.

DMR Problem Analysis

For 2D problem, we use a slice-by-slice approach to detect shocks. Take the DMR problem as an example:



The method detects shocks successfully ($N_x\times N_y=800\times 200$, 4 sub-domains and 15 overlapping points in X and Y).

Future Work

Improve the efficiency of IAMQ-RBF and consider Euler equation with non-uniform grid system.

References

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